Beam-Beam Simulation For Hadron Colliders

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Thanks: **W. Herr, M. Vogt, J. Qiang,**
K. Ohmi, A. Kabel, S. Tzenov,
....

OUTLINE

- Some Basic Characteristics of Beam-Beam Interactions of Hadron Beams
- Recent Numerical Results and Experimental Observations
- Methods of Beam-Beam Simulation for Hadron Beams
- Final Comments

Some Comments on Studies of Beam-Beam Effects of Hadron Beams

The motion of particles is a Hamiltonian dynamics.

With nonlinear perturbations, particle distributions may not reach any stationary state within a fraction of the storage time.

The problem of beam-beam interactions can be divided into near-linear (near-integrable) and nonlinear (non-integrable) regime based on the validity of linearized (or perturbative) Vlasov equation.

Near-Linear (Near-Integrable) Regime:

- **Quasi-stationary states of Vlasov equation may exist, especially when $\xi \longrightarrow 0$.**
- **Methods of perturbation could be employed.**
- **The system is forgiving on methods of numerical simulation.**
- **In principle, beams are stable in the consideration of beam-beam interactions and emittance growth is not important (or significant) after an initial beam filementation.**

Nonlinear (Nonintegrable) Regime:

- **No stationary state for Vlasov equation.**

  $\Longrightarrow$ We have to work with transient states of a nonlinear PDE — a very tough problem mathematically.

- **Methods of perturbation such as various canonical perturbation expansions, the truncation of moment expansions, or the linear-stability analysis of steady states of Vlasov equation are no longer valid. The use of those approximation methods could distort the dynamics.**

  $\Longrightarrow$ Only validated method is a correct numerical simulation.

- **Fine Hamiltonian structure in phase space is important.**

  $\Longrightarrow$ For a correct beam-beam simulation:

  Need to calculate a "smooth" and "undistorted" beam-beam force;

  In order to sample enough detail of phase-space structure for the time scale of interest, a large number of macro-particles are necessary.

- **Be careful to use classical diffusion models for emittance growth or beam-particle loss. They are only valid mathematically in a fully chaotic region, otherwise the stickiness of resonances results in non-$\delta(\tau)$ correlations — the problem of long-term tails.**

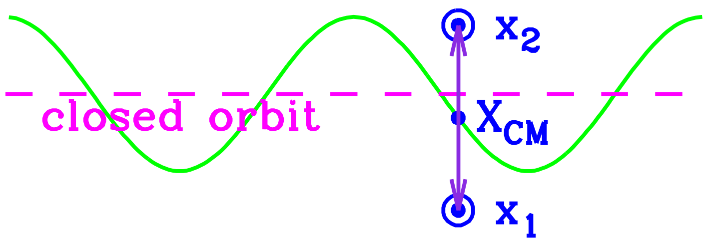
Coherent Beam-Beam Oscillation

Traditionally,

Strong-Strong Beam-Beam Effect

$\Longleftrightarrow$ **Coherent Beam-Beam Effect**

Motion Of Beams With Beam-Beam Interaction:



$$\vec{X}_{CM} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2}{m_1 + m_2}$$

$$\Delta\vec{x} = \vec{x}_1 - \vec{x}_2$$

σ-mode:

$\vec{X}_{CM}$ **does not feel the beam-beam force and oscillates with betatron tunes.**

π-mode, From Linear Theory:

$\Delta\vec{x}$ **oscillates with betatron tunes plus a coherent beam-beam tune shift.**

- **For symmetrical (round to flat) beams, coherent beam-beam tune shift is $\sim 1.2 - 1.4\ \xi$ (Yokoya factor).**

- **For unsymmetrical beams, when the ratio of ξ is less than ~ 0.55 or when difference between betatron tunes of two beams is larger than ξ, the π-mode would be damped.**

$\Longrightarrow$ **LINEAR THEORY: Coherent beam-beam effect is not important in the unsymmetrical or strong-weak cases.**

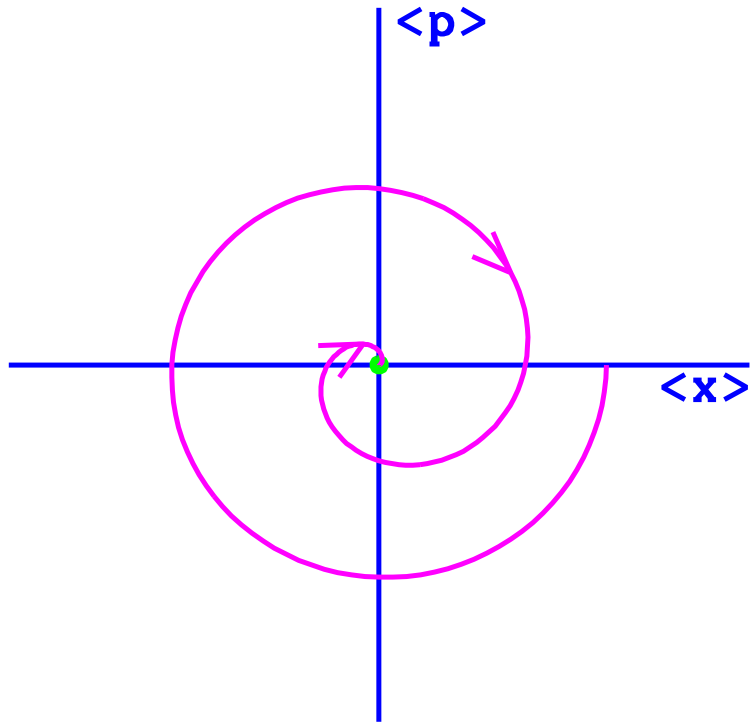
This is not right in the nonlinear regime of beam-beam interactions of hadron beams!

Coherent Beam-Beam Instability of Lepton Beams

A.W. Chao and R.D. Ruth

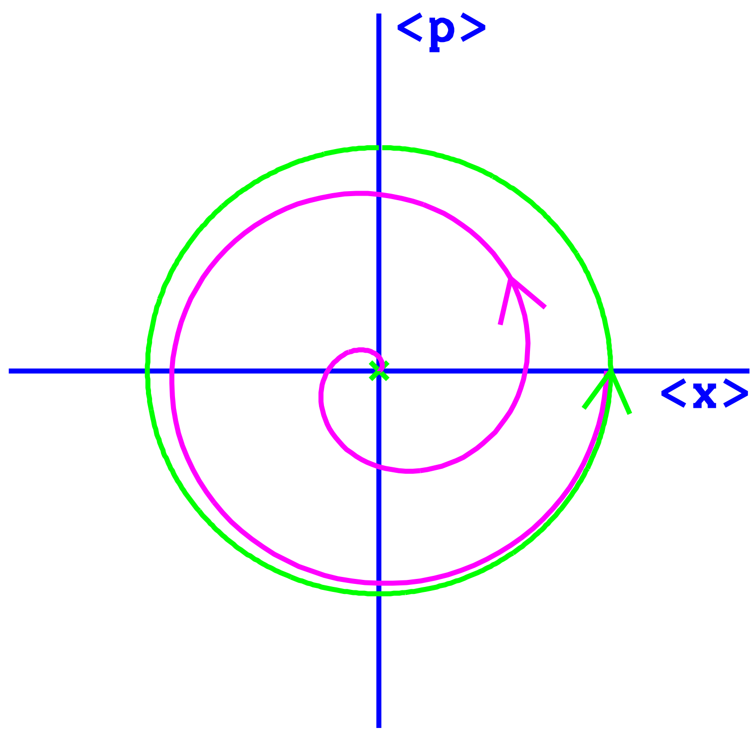
$\xi < \xi_c$,

- The origin of the phase space for beam-centroid motion is a stable fixed point.
- Damped coherent oscillation due to radiation damping.



$\xi > \xi_c$,

- The origin of the phase space for beam-centroid motion is an unstable fixed point.
- The competition between the instability and the damping could result in stable π or high-order modes.



Coherent Beam-Beam Instability of Hadron Beams

Regular Coherent Oscillation When $\xi < \xi_c$

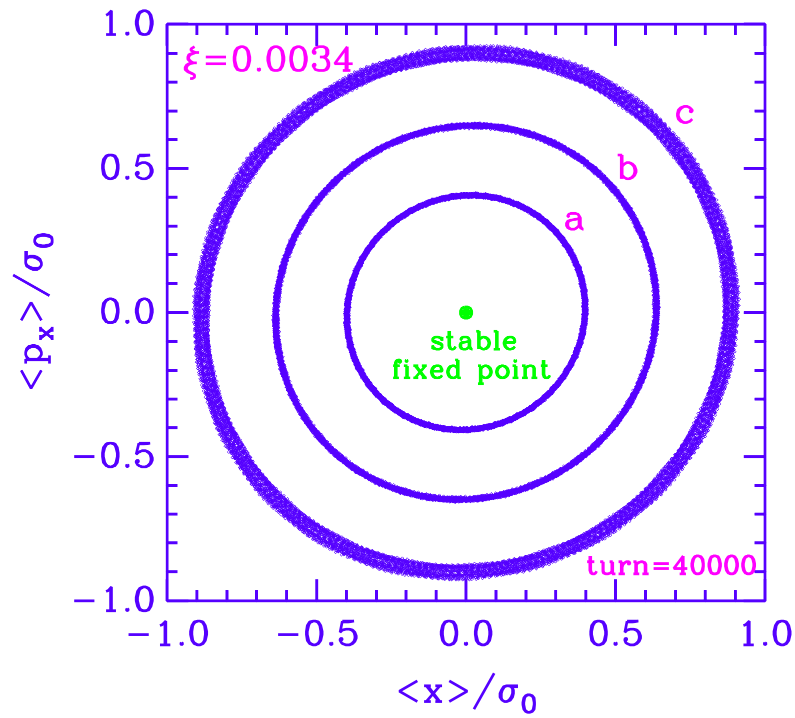
The origin of phase space is stable for coherent oscillation.

Symmetrical Beams:

- Coherent oscillations are stable.
- Yokoya factor is valid when ξ is small.

Unsymmetrical Beams:

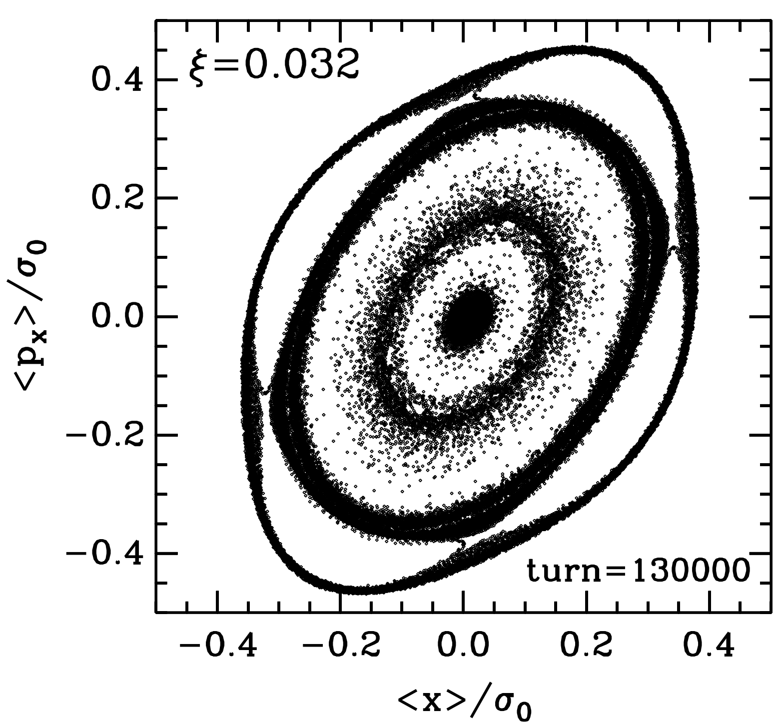
- Landau damping could suppress coherent motions but result in a emittance increase.



Chaotic Coherent Oscillation When $\xi > \xi_c$

The origin of phase space is unstable for coherent motion.

- Coherent oscillations are chaotic.
- Onset of collective beam-beam instability due to the chaotic motion.
- Collective beam-beam instability could occur with both strong-strong or strong-weak beam-beam interactions.



Collective Beam-Beam Instability of Hadron Beams

When the beam-beam parameter (ξ) exceeds a threshold (ξ_c), a chaotic coherent beam-beam instability occurs with the following characteristics:

- **Chaotic Coherent Oscillation**

  The phase-space region nearby the closed orbit could be unstable for beam centroids.

  $\Longrightarrow$ Spontaneous Chaotic Coherent Oscillation

- **Emittance Growth**

  An enhanced emittance growth is due to the dynamics of the counter-rotating beam.

- **Formation of Beam Halo**

  Beam distributions could significantly deviate from a Gaussian due to beam halo. The formation of the beam halo is a result of chaotic transport of particles from beam cores to beam tails.

[Ref.: J. Shi & D. Yao, PRE 62, 1258 (2000)]

Collective Beam-Beam Instability in Case of Strong-Weak Beam-Beam Interactions

- **The self-consistent beam-beam simulation predicted that the chaotic coherent beam-beam instability could occur in HERA Upgrade. The onset of the instability is due to an overlap of the electron beam (weak beam) with the 4th-order beam-beam resonance.**

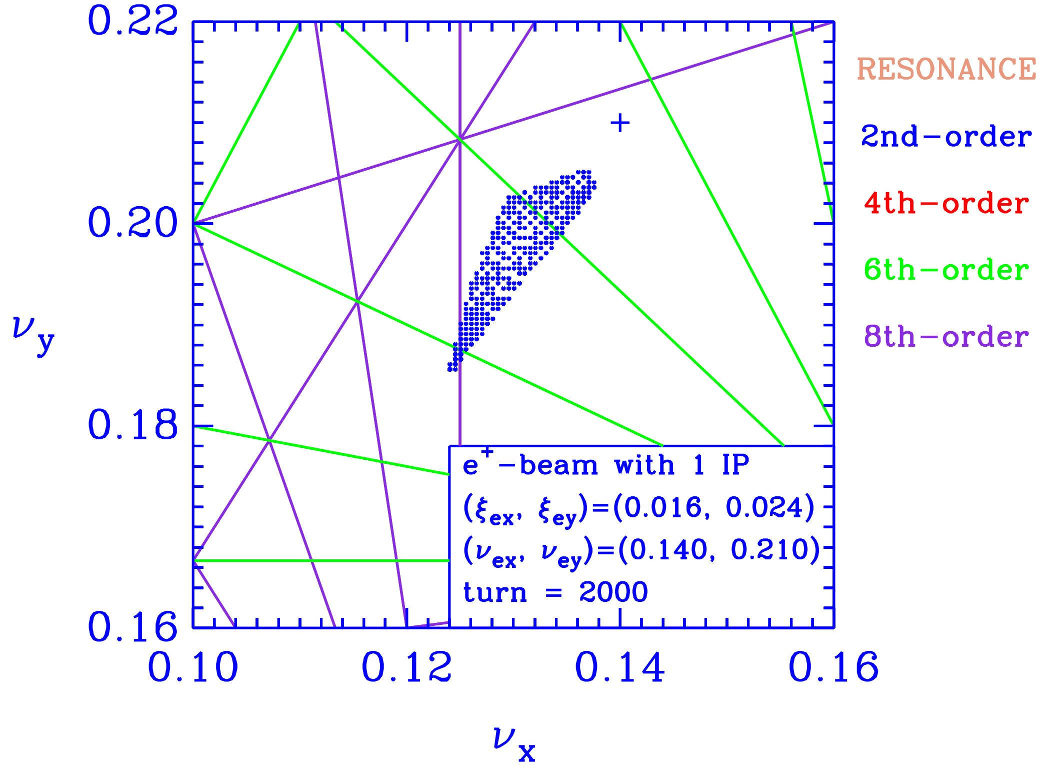
- **Such the collective beam-beam instability in HERA has been confirmed by experiments on HERA recently. The phenomena observed in the experiments remarkably agree with the prediction.**

In HERA, $\xi_{e,x}/\xi_{p,x} \sim 20, \quad \xi_{e,y}/\xi_{p,y} \sim 100$

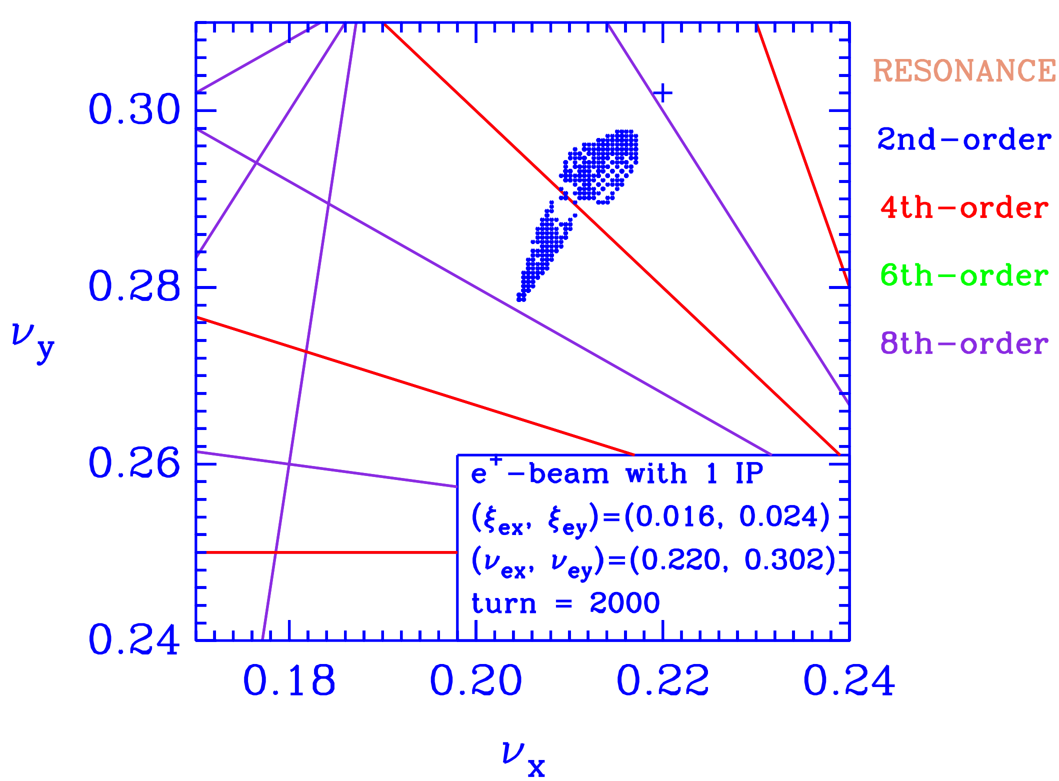
$\Longrightarrow$ **It is a typical strong-weak beam-beam interaction!**

HERA 2003 STUDY: Tune Spread of e^+ Beam (1 IP)

The e^+ beam is at nominal working point :

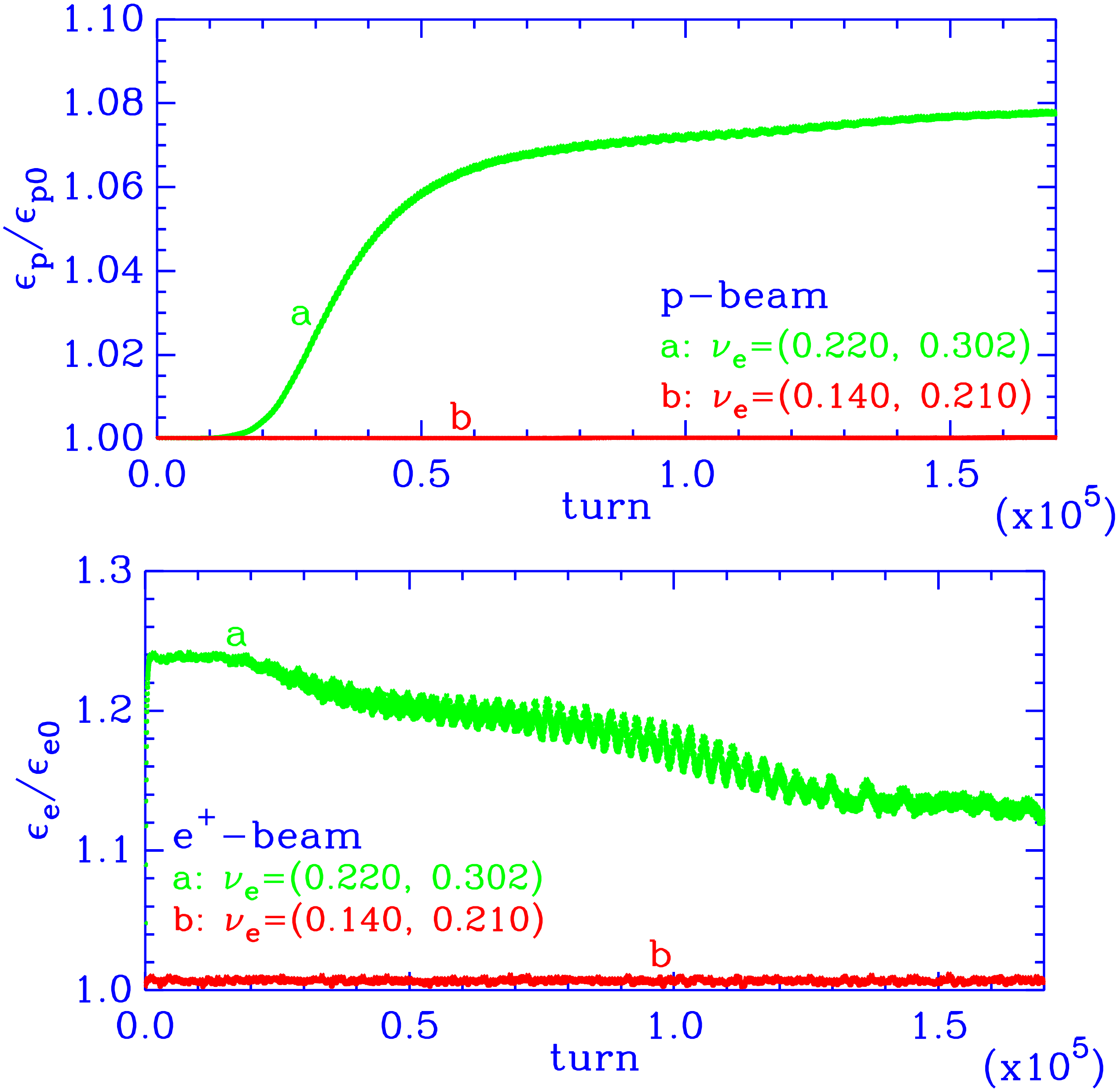


The e^+ beam crosses $2\nu_x + 2\nu_y$=1 :



HERA 2003 High-Luminosity Study With One IP

Emittance Growth due to Collective Beam-Beam Instability



HERA 2003 Experimental Result:
In case a, the proton beam emittance increases ∼30% while in case b, no emittance increase was observed.

Methods of Beam-Beam Simulation

1. **Soft Gaussian approximation: Assume Gaussian beams with varying width and center.**

   — Fast [$O(N_p)$]; but not right in the nonlinear regime of beam-beam interactions in which the distribution could deviate from the Gaussian; may be o.k. for incoherent beam-beam effects.

2. **Direct multi-particle tracking: the beam-beam force is calculated with particles-to-particle individually.**

   — Precise if N_p is large, but very slow [$\mathrm{O}(N_p^2)$],
   typical: $N_p \leq 10^4 \Longrightarrow$ wrong physics in the nonlinear regime.

3. **Particle-In-Cell (PIC): evaluate beam-beam force on a mesh.**

   — Precise, but very slow for separated beams.

   Variations:

   a. Calculate Beam-Beam Potential Without Boundary

   b. Calculate The Potential With Approximated Boundary

   c. Directly Calculate Beam-Beam Force on the Mesh

   d. With Weighted Functions

4. **Hybrid Fast Multipole Method (HFMM)**

   — Fast, better for separated beams.

5. **Canonical perturbations for solving Vlasov equation**

   — Only valid for $\xi \longrightarrow 0$.

Field Computation With PIC Method

1. Solve Beam-Beam Potential on the Mesh

— Computation cost: $N_p N_m \ln N_m$

Poisson eq. for potential $\Phi(x, y)$ with charge density $\rho(x, y)$,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \Phi(x, y) = -2\pi\rho(x, y)$$

With Green's function,

$$\Phi(x, y) = \int G(x - x', y - y')\rho(x, y)dx'dy'$$

For open boundary,

$$G(x, y) = -\frac{1}{2} \ln\left(x^2 + y^2\right)$$

FFT is usually used for solving $\Phi(x, y)$ on the mesh.
The field is then computed with numerical derivatives.

Comment: Fast; but the mesh has to be big to minimize errors from the boundary. Many empty cells are wasted.

2. Direct Calculation of Beam-Beam Field on the Mesh

— Computation cost: $N_p N_m^2$

The field is calculated with

$$\vec{K}(\vec{r}) = \int d\vec{r'}\rho(\vec{r'})\vec{G}_k(\vec{r} - \vec{r'})$$

where Green's function is

$$\vec{G}_k(\vec{r} - \vec{r'}) = \frac{(\vec{r} - \vec{r'})}{(x - x')^2 + (y - y')^2}$$

Comment: Accurate; only a small number of empty cells when using adaptive mesh; slow when a large mesh has to be used (mismatch beams).

Weighted Macro-Particles (WMPT)

—— M. Vogt, J.A. Ellison, T. Sen, R.L. Warnock

For any function in phase space $A(\vec{z})$,

$$\langle A \rangle_t = \int A(\vec{z}) f(\vec{z}, t) d^4 z$$

where $f(\vec{z}, t)$ is the beam distribution in phase space. Because of the symplecticity,

$$\langle A \rangle_t = \int A(\vec{z}(t)) f(\vec{z}(0), 0) d^4 z(0)$$

On grid points with weighted function w_i,

$$\langle A \rangle_t = \sum_i A(\vec{z}_i(t)) f(\vec{z}_i(0), 0) w_i$$

Advantage: better sampling beam tails.

Hybrid Fast Multipole Method (HFMM)

—— W. Herr, M.P. Zorzano, and F. Jones

The field is calculated on a mesh:

- Macro-particles inside the grid are assigned to grid points;
- Multipole expansions of the field are computed on every grid points.

Computing cost: Between $O(N_m)$ and $O(N_m \log N_m)$.

A better way to treat long-range beam-beam interactions.

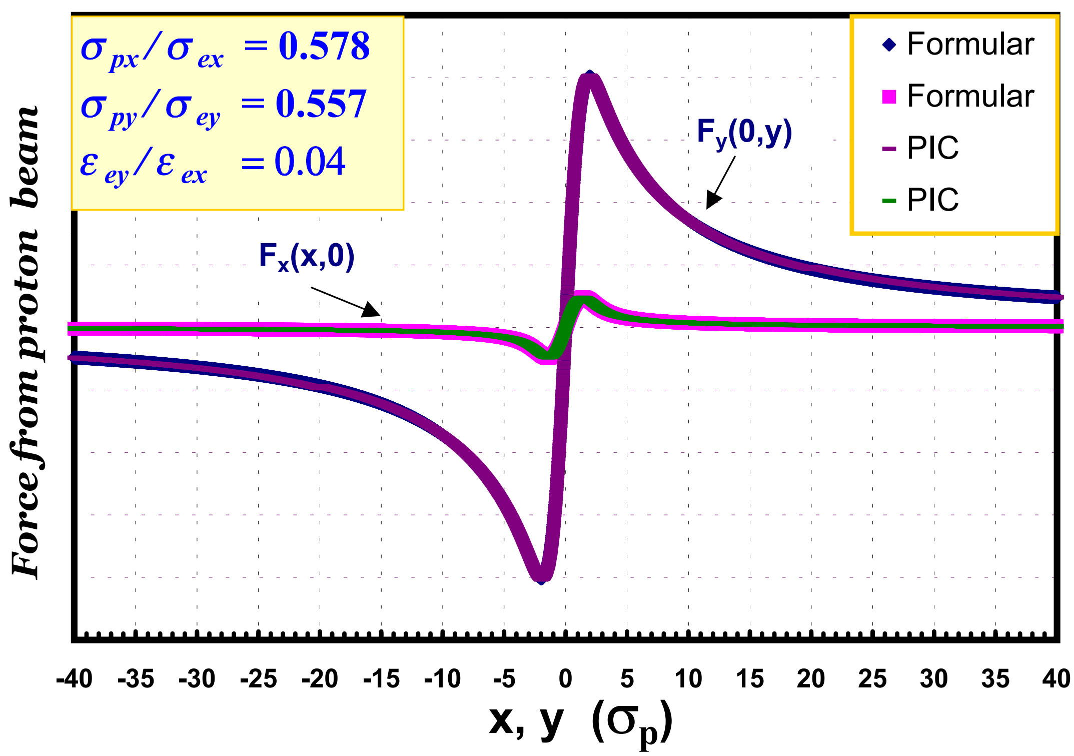
The Correct Way of Beam-Beam Simulation

All computational parameters in a numerical model should be tested for the computational convergence for the system in the worst possible situation (maximal beam-beam parameter, worst working point, ...).

—— A code should never be made as a "one-size-fits-all".

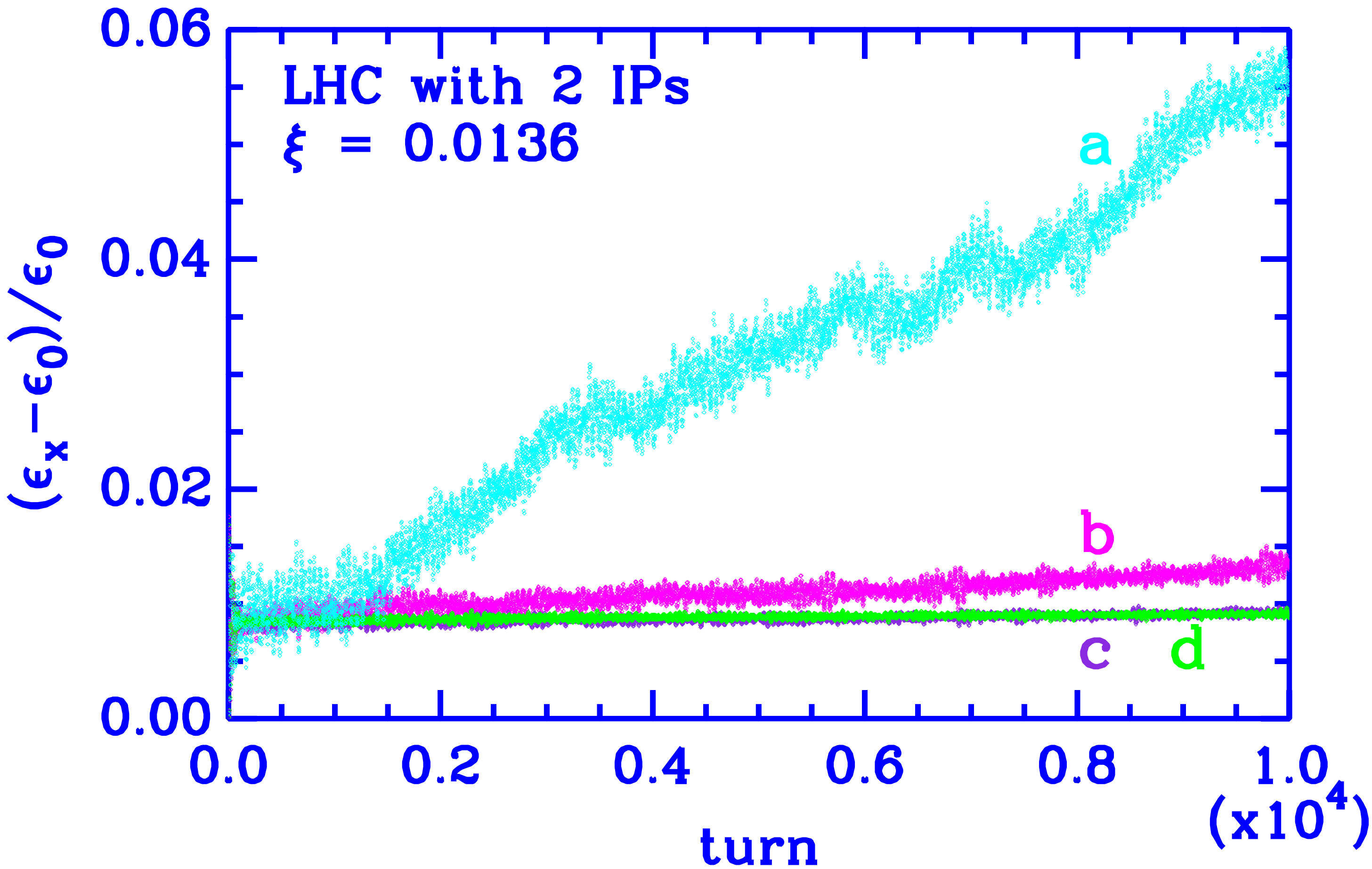
Importance of Computational Convergence

Traditionally, the "beauty" of the initial field has been used to show how "good" a simulation is,



This is far from enough especially in the nonlinear regime of beam-beam interactions.

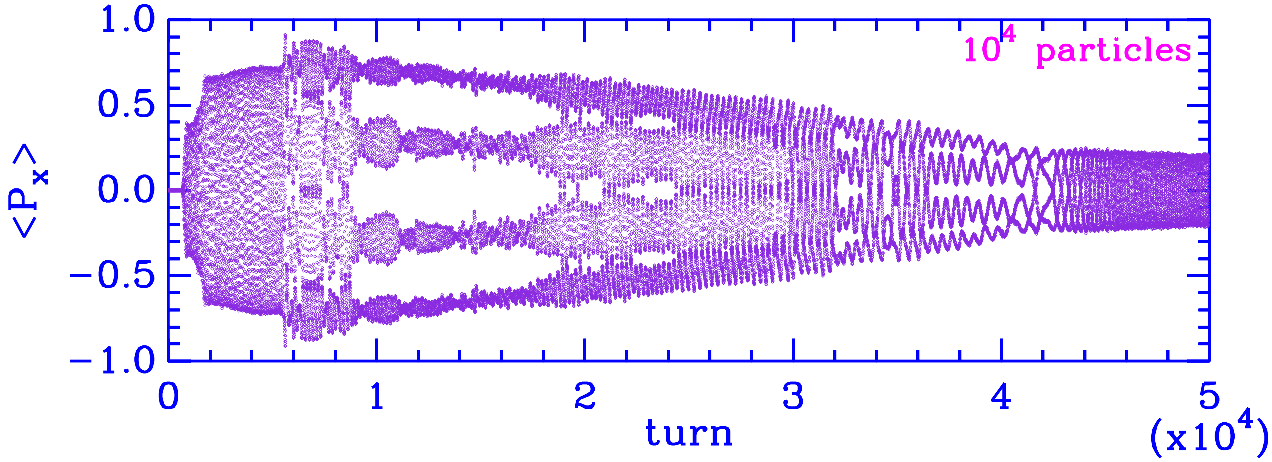
Comparison Between Different Numbers of Macro-Particles

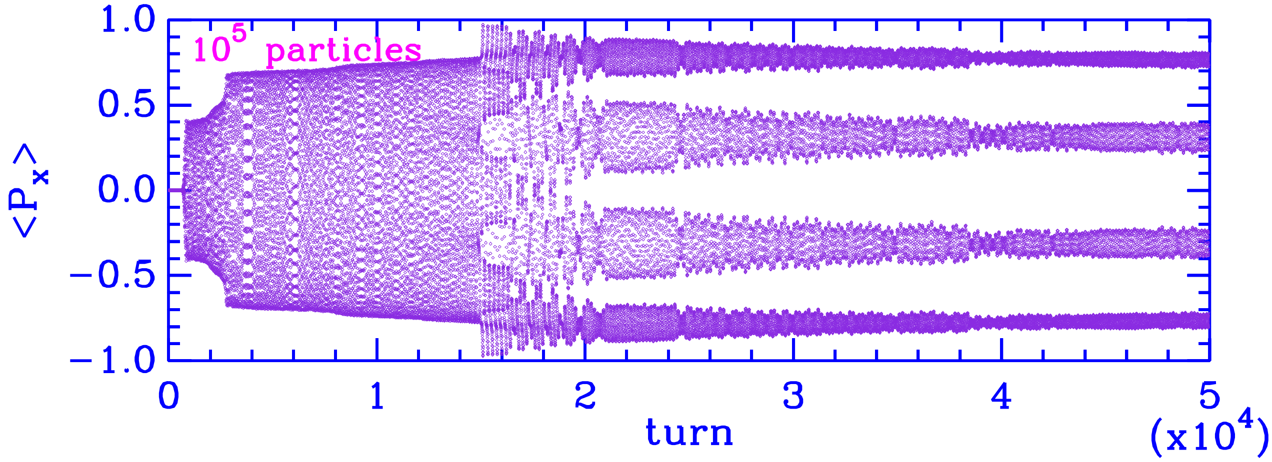


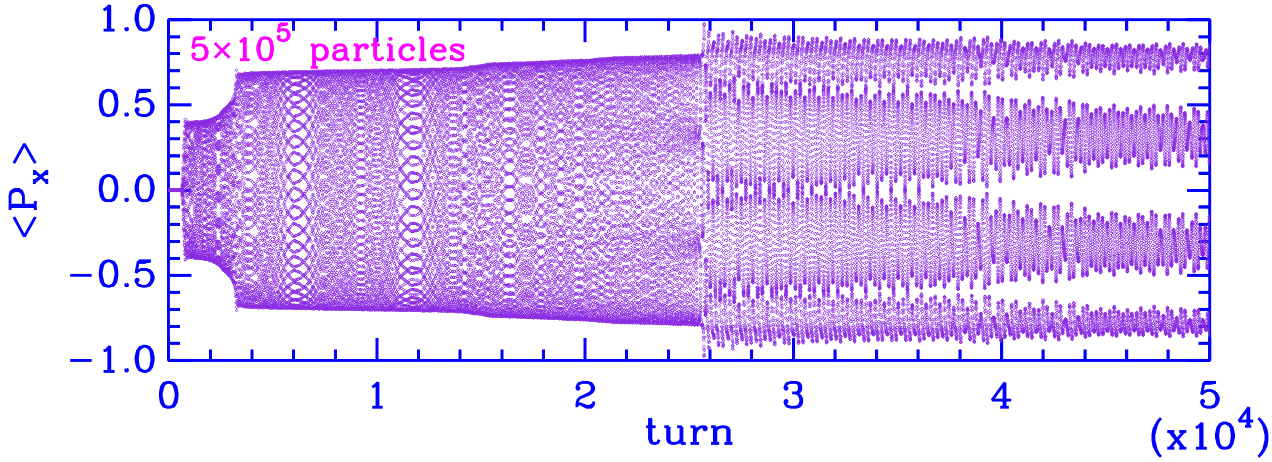
(a) 10^4 particles; (b) 10^5 particles;
(c) 5×10^5 particles; (d) 10^6 particles.

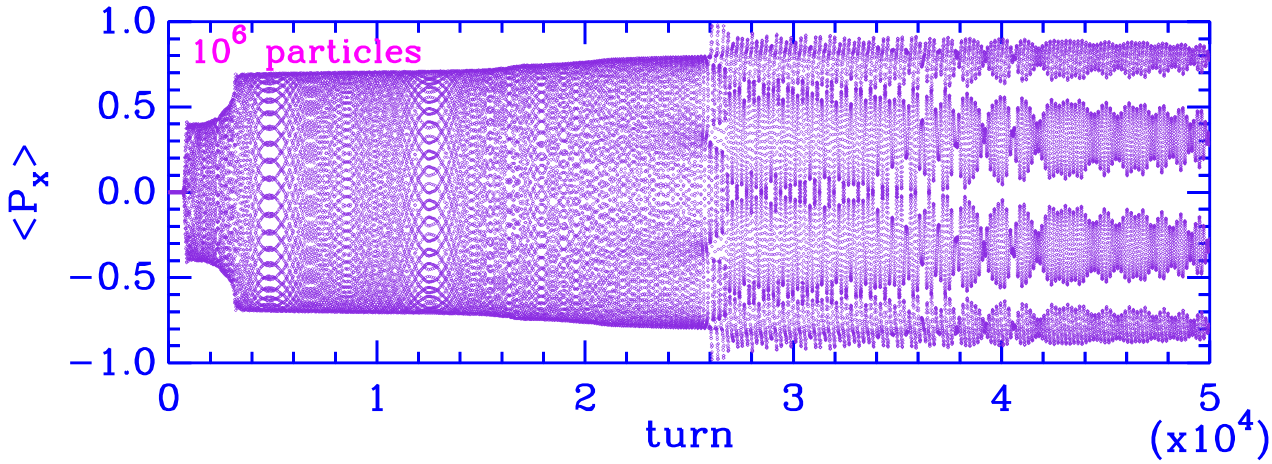
$$\epsilon_x = \int \frac{1}{2}(x^2 + p_x^2) f(\vec{r}, \vec{p}, t) d\vec{r} d\vec{p}$$

Comparison Between Different Numbers of Macro-Particles

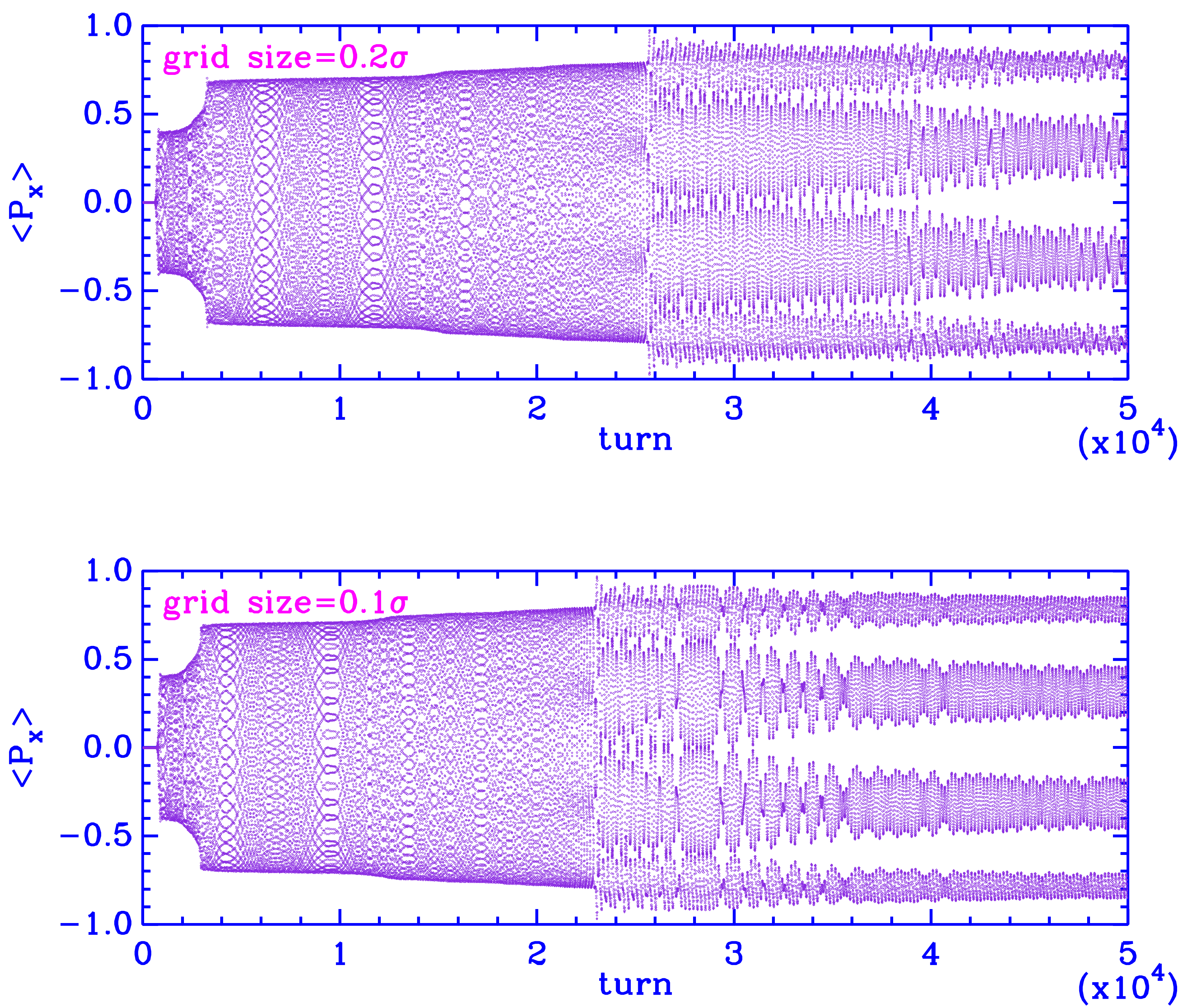








COMPARISON BETWEEN DIFFERENT GRID CONSTANTS



Final Comments

- **To push the frontier of luminosity, hadron colliders could be more likely operated in the nonlinear regime of beam-beam interactions. An understanding of beam-beam effects in that regime is necessary.**

- **To study the beam-beam effects, especially in the nonlinear regime, we have to respect the Hamiltonian nature of hadron beams, and we have to recognize that the traditional mode analysis based on the linearized Vlasov equation, which is a very useful tool in lepton colliders, is invalid for hadron beams mathematically.**

- **In the nonlinear regime of beam-beam interactions, the traditional boundary between strong-strong and strong-weak beam-beam interactions is blurred and the beam-beam effect has to be studied (or at least checked) self-consistently in all situations. In this regime, only validated method for the study of nonlinear beam-beam effect is numerical simulation.**

- **What We Can Do Computationally**
  Understanding of Short-term beam-beam effects:
    Fast emittance growth (within $\sim 10^6$ tunes)
    Onset of beam-beam instabilities
    ...

- **What We Don't Have Confident Computationally**
  Understanding of Long-term beam-beam effects:
    Slow emittance growth, slow particle loss, and
    Slow diffusion due to nonlinearities
    Beam lift time ?